A Smoothing Method of Discrete Breakup S-matrix Elements in the Theory of Continuum-Discretized Coupled Channels

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We present a practical way of smoothing discrete breakup S-matrix elements calculated by the continuum-discretized coupled-channel method (CDCC). This method makes the smoothing procedure much easier. The reliability of the smoothing method is confirmed for the three-body breakup reactions, ^{58}Ni(d,pn) at 80 MeV and ^{12}C(^{6}He,^{4}He^{2}n) at 229.8 MeV.

§1. Introduction

Secondary beam experiments are opening up new physics on unstable nuclei away from the stability line in the nuclear chart. Such unstable nuclei have exotic properties, e.g., the halo structure.[1)–3)] In scattering of such a weakly bound projectile, it easily breaks up into its constituents, and hence the reaction should be described as at least a three-body scattering.

One of the most reliable methods for treating the projectile breakup processes in the wide range of incident energy is the method of continuum-discretized coupled channels (CDCC).[4),5)] In CDCC, the scattering wave function of the total system is expanded with the complete set of bound and continuum states of the projectile. The continuum states are classified by linear and angular momenta, k and ℓ, and truncated at upper bounds, $k_{\max}$ and $\ell_{\max}$, respectively. This truncation is based on the assumption that the breakup processes take place in the truncated space. The k-continuum is divided into small bins and the continuum states in each bin are averaged into a single state. This discretization procedure is called the average discretization method. The space spanned by the bound states and the finite number of discretized continuum states are called the modelspace. The S-matrix elements calculated with CDCC converge as the modelspace is extended.[6),7)] The converged CDCC solution is the unperturbed solution of the distorted Faddeev equations, and corrections to the solution are negligible within the spatial region in which the breakup processes take place.[8),9)]

The average discretiztation method has widely been used so far, but its application is limited in the three-body breakup processes in scattering of two-body projectiles. The average method is not feasible for the four-body breakup processes in scattering of three-body projectiles, since it requires exact three-body continuum states of the projectile that are quite difficult to obtain. This problem can be solved by using the pseudostate discretization method[10)–16)] in which the continuum states $\{\psi(\boldsymbol{k})\}$ are replaced by pseudostates $\{\hat{\Phi}_i\}$ obtained by diagonalizing the internal Hamiltonian of the projectile in a space spanned by L^2-type basis functions. One can adopt the Gaussian[10),11)] or the transformed harmonic oscillator (THO)[12)] ba-

sis as the L^2-type basis functions. The validity of the pseudostate discretization method was confirmed in scattering of two-body projectile by the good agreement between the CDCC solutions obtained by the pseudostate discretization and the average methods.[10),11),13)–15)]

For four-body reaction systems, CDCC calculations with the pseudostate discretization method based on Gaussian[13)–15)] or THO[16)] basis functions were successful in describing the elastic scattering at not only high energies but also low energies near the Coulomb barrier. Thus, the back-coupling effects of four-body breakup processes on the elastic scattering, i.e., *virtual* four-body breakup processes, are well described by CDCC based on the pseudostate discretization method.

The S-matrix elements calculated with CDCC, $\hat{S}_i$, are discrete in $\boldsymbol{k}$, while the exact ones $S(\boldsymbol{k})$ are continuous. Thus, one needs a way of smoothing $\hat{S}_i$ in order to analyze real breakup processes, i.e. breakup reactions themselves. In principle, this can be possible by calculating the overlap $\mathcal{F}_i(\boldsymbol{k}) = \langle\psi(\boldsymbol{k})|\hat{\Phi}_i\rangle$ between the exact continuum states $\psi(\boldsymbol{k})$ and the pseudostates $\hat{\Phi}_i$; see §2.2 for the detail. In practice, this smoothing factors $\mathcal{F}_i(\boldsymbol{k})$ can easily be obtained in three-body breakup reactions of two-body projectiles, but it is quite hard in four-body ones of three-body projectiles. Very recently, we have proposed a smoothing method applicable to four-body breakup reactions,[17)] but it still requires heavy numerical calculations and hence not so practical. Thus, it is highly expected that an accurate and practical method of smoothing $\hat{S}_i$ is proposed.

The complex scaling method (CSM)[18)] is a powerful tool of solving many-body resonance and weakly bound states. Recently, CSM was extended so that it can treat the electromagnetic transition of the core+nucleon+nucleon system such as ^{6}He and ^{11}Li from its ground state to the continuum ones.[19),20)] As a merit of CSM, the calculated transition strength is obtained as a continuum spectrum. This implies that an accurate and practical method of smoothing $\hat{S}_i$ can be constructed with CSM.

In this Letter, we present a simple formula that makes it possible to smooth $\hat{S}_i$ accurately, using CSM. This formula is useful for not only three-body but also four-body breakup reactions. Validity of the formula is tested in the three-body breakup reactions, ^{58}Ni(d,pn) at 80 MeV and ^{12}C(^{6}He,^{4}He^2n) at 229.8 MeV, since in such three-body reactions the "exact" S-matrix elements are obtainable by calculating $\mathcal{F}_i(\boldsymbol{k})$ exactly. This test is inevitable to proceed to CDCC calculations for four-body breakup processes in future.

This Letter is constructed as follows. In Sec. 2, we first recapitulate CDCC and the pseudostate discretization method, and present an accurate and practical smoothing formula of $\hat{S}_i$. Test calculations are done in Sec. 3. Section 4 gives a summary.

§2. Formulation

2.1. *CDCC and the pseudostate discretization*

For simplicity, we consider the three-body (A+b+c) system, where particles A, b and c are assumed to be structureless. It is straightforward to extend the present formulation to the four-body system in which the projectile consists of three constituents. The total Hamiltonian of the A+b+c system is

$$H = T_R + U + H_{\rm B}, \tag{1}$$

$$U = U_b(\boldsymbol{R}_b) + U_c(\boldsymbol{R}_c) + V^{\rm Coul}(\boldsymbol{R}), \tag{2}$$

$$H_{\rm B} = T_r + V_{bc}, \tag{3}$$

where $\boldsymbol{R}$ is the relative coordinate between the center-of-mass (c.m.) of the projectile B=b+c and A. Coordinate $\boldsymbol{R}_{\rm x}$ (x = b and c) denotes the relative coordinate between x and A, and T_ξ ($\xi = \boldsymbol{R}$ and $\boldsymbol{r}$) is the kinetic-energy operator associated with ξ. Interaction $V_{\rm bc}$ is the potential between b and c, and $U_{\rm x}$ is the nuclear part of the optical potential between x and A. Meanwhile the Coulomb interaction $V^{\rm Coul}$ between B and A is treated approximately as a function of $\boldsymbol{R}$ only, i.e., we neglect Coulomb breakup processes.

In the pseudostate discretization method,[10),11),13)–15)] the pseudostates $\{\hat{\Phi}_i\}$ are obtained by diagonalizing the internal Hamiltonian $H_{\rm B}$ in a space spanned by the L^2-type Gaussian basis functions:[21)]

$$\langle\hat{\Phi}_i|H_{\rm B}|\hat{\Phi}_{i'}\rangle = \delta_{ii'}\hat{\varepsilon}_i, \tag{4}$$

where $\hat{\varepsilon}_i$ is the eigenenergy of $\hat{\Phi}_i$. The subscript i denotes a set of quantum numbers, i.e. the energy index n of the pseudostates, the angular momentum ℓ between b and c, and its projection on the z-axis m. The eigenstates of $H_{\rm B}$ with negative and positive energies correspond to the bound state(s) and the pseudostates, respectively, and the latter are regarded as discretized continuum states.

The basic assumption of CDCC is that the breakup processes take place in a model space[14),15)]

$$\mathcal{P} = \sum_i |\hat{\Phi}_i\rangle\langle\hat{\Phi}_i|. \tag{5}$$

Validity of this assumption is justified by the fact that the calculated elastic and total breakup cross sections of the three- and four-body scatterings converge as the modelspace is extended.[10),11),13)–15)] Thus, one may regard $\{\hat{\Phi}_i\}$ as a complete set in describing the reaction processes considered. We henceforth call $\{\hat{\Phi}_i\}$ the approximate complete set in this meaning.

We solve the three-body Schrödinger equation in the modelspace $\mathcal{P}$,

$$\mathcal{P}[H - E_{\rm tot}]\mathcal{P}|\Psi^{\rm CDCC}\rangle = 0, \tag{6}$$

where $E_{\rm tot}$ is the total energy of the system. The total wave function $\Psi^{\rm CDCC}$ is

expanded by the approximate complete set $\{\hat{\Phi}_i\}$:

$$|\Psi^{\rm CDCC}\rangle = \sum_i |\hat{\Phi}_i, \hat{\chi}_i\rangle, \tag{7}$$

where $|\hat{\Phi}_i, \hat{\chi}_i\rangle = |\hat{\Phi}_i\rangle \otimes |\hat{\chi}_i\rangle$ and $i = 0$ denotes the elastic channel and others ($i \neq 0$) the breakup channels. The expansion coefficient $|\hat{\chi}_i\rangle$ describes the relative motion between B (in state $\hat{\Phi}_i$) and A. The intrinsic energy $\hat{\varepsilon}_i$ of B and the relative momentum $\hat{P}_i$ between B and A satisfy the energy conservation, $E_i \equiv \hbar^2 \hat{P}_i^2/(2\mu) = E_{\rm tot} - \hat{\varepsilon}_i$, with the reduced mass μ between B and A.

Multiplying Eq. (6) by $\langle\hat{\Phi}_i|$ from the left, we can obtain a set of coupled differential equations for $|\hat{\chi}_i\rangle$, called CDCC equation,

$$[T_R + \hat{U}_{i,i} - E_i]|\hat{\chi}_i\rangle = -\sum_{i' \neq i} \hat{U}_{i,i'}|\hat{\chi}_{i'}\rangle. \tag{8}$$

The coupling potential $\hat{U}_{i,i'}$ is defined by

$$\hat{U}_{i,i'} = \langle\hat{\Phi}_i|U|\hat{\Phi}_{i'}\rangle. \tag{9}$$

The CDCC equation (8) is solved under the usual boundary condition for $\langle \boldsymbol{R}|\hat{\chi}_i\rangle \equiv \hat{\chi}_i(\boldsymbol{R})$.[4),5)]

2.2. *Smoothing method*

In this subsection, we present a new way of smoothing discrete breakup S-matrix elements $\hat{S}_i$ obtained by CDCC. The exact breakup T-matrix element to the exact continuum state $\psi(\boldsymbol{k})$ of B is given by

$$T^{\rm EX}(\boldsymbol{k}, \boldsymbol{P}) = \langle\psi(\boldsymbol{k}), P|U|\Psi\rangle \tag{10}$$

with $|\psi(\boldsymbol{k}), P\rangle = |\psi(\boldsymbol{k})\rangle \otimes |P\rangle$, where $\boldsymbol{k}$ and $\boldsymbol{P}$ are momenta in the asymptotic region associated with the coordinates $\boldsymbol{r}$ and $\boldsymbol{R}$, respectively; $|\psi(\boldsymbol{k})\rangle$ is the exact two-body wave function of B with energy ε satisfying

$$[H_{\rm B} - \varepsilon]|\psi(\boldsymbol{k})\rangle = 0, \tag{11}$$

and $|P\rangle$ is the plane wave function satisfying

$$[T_R - (E_{\rm tot} - \varepsilon)]|P\rangle = 0. \tag{12}$$

The exact three-body (A+b+c) wave function Ψ can be replaced by the corresponding CDCC wave function $\Psi^{\rm CDCC}$ with good accuracy. Inserting the approximate complete set $\mathcal{P}$ of Eq. (5) between the bra-vector and the operator U of the right hand side of Eq. (10), we can obtain the approximate smooth T-matrix elements $T(\boldsymbol{k}, \boldsymbol{P})$:

$$\begin{aligned} T(\boldsymbol{k}, \boldsymbol{P}) &= \sum_i \langle\psi(\boldsymbol{k})|\hat{\Phi}_i\rangle\langle\hat{\Phi}_i, \hat{P}_i|U|\Psi^{\rm CDCC}\rangle \\ &\equiv \sum_i \mathcal{F}_i(\boldsymbol{k})\hat{T}_i, \end{aligned} \tag{13}$$

where $|\hat{\Phi}_i, \hat{P}_i\rangle = |\hat{\Phi}_i\rangle \otimes |\hat{P}_i\rangle$, $\mathcal{F}_i(\boldsymbol{k})$ is the smoothing factor defined by

$$\mathcal{F}_i(\boldsymbol{k}) = \langle \psi(\boldsymbol{k}) | \hat{\Phi}_i \rangle, \tag{14}$$

and $\hat{T}_i$ is the breakup T-matrix element of CDCC defined by

$$\hat{T}_i = \langle \hat{\Phi}_i, \hat{P}_i | U | \Psi^{\mathrm{CDCC}} \rangle. \tag{15}$$

Since the breakup T-matrix elements are proportional to the breakup S-matrix elements, Eq. (13) is reduced to

$$S(\boldsymbol{k}, \boldsymbol{P}) = \sum_i \mathcal{F}_i(\boldsymbol{k}) \hat{S}_i. \tag{16}$$

The smoothing factor $\mathcal{F}_i(\boldsymbol{k})$ is the overlap between the pseudostate $\hat{\Phi}_i$ and the exact continuum state $\psi(\boldsymbol{k})$. We propose to use the complex-scaling method (CSM)[18] for the calculation of $\mathcal{F}_i(\boldsymbol{k})$. This new smoothing method is applicable to not only the two-body projectile but also the three-body one, as shown below.

In CSM, the scaling transformation operator $C(\theta)$ and its inverse operator are defined by

$$\langle \boldsymbol{r} | C(\theta) | f \rangle = e^{i3/2\theta} f(\boldsymbol{r} e^{i\theta}), \quad \langle g | C^{-1}(\theta) | \boldsymbol{r} \rangle = \{ e^{-i3/2\theta} f(\boldsymbol{r} e^{-i\theta}) \}^*, \tag{17}$$

since

$$\langle g | C^{-1}(\theta) C(\theta) | f \rangle = \int d\boldsymbol{r} \langle g | C^{-1}(\theta) | \boldsymbol{r} \rangle \langle \boldsymbol{r} | C(\theta) | f \rangle = \langle g | f \rangle \tag{18}$$

for any normalizable states g and f, and hence $C^{-1}(\theta)C(\theta) = 1$. The transformed Scrhödinger equation is defined by

$$(H_{\mathrm{B}}^{\theta} - \varepsilon^{\theta}) | \psi^{\theta} \rangle = 0, \quad \langle \tilde{\psi}^{\theta} | (H_{\mathrm{B}}^{\theta} - \varepsilon^{\theta}) = 0 \tag{19}$$

with

$$\langle \boldsymbol{r} | H_{\mathrm{B}}^{\theta} | \boldsymbol{r}' \rangle = \langle \boldsymbol{r} | C(\theta) H_{\mathrm{B}} C^{-1}(\theta) | \boldsymbol{r}' \rangle = \left[-e^{-2i\theta} \frac{\nabla_{\boldsymbol{r}}^2}{2\mu_{bc}} + V_{bc}(\boldsymbol{r} e^{i\theta}) \right] \delta(\boldsymbol{r} - \boldsymbol{r}'), \tag{20}$$

where μ_{bc} is the reduced mass between b and c. The transformed Hamiltonian H^{θ} is not hermitian when $\theta \neq 0$ and thus the ket vectors $|\psi^{\theta}\rangle$ and the bra ones $\langle \tilde{\psi}^{\theta}|$ are biorthogonal to each other.[22] Properties of $|\psi^{\theta}\rangle$ and ε^{θ} are summarized as follows.

1. In the case that $|\psi^{\theta}\rangle$ is a bound state $|\psi_j^{\theta}\rangle$ with a negative energy $\varepsilon_j^{\theta} = -\hbar^2 (k_j^{\theta})^2 / (2\mu_{bc})$, $\langle \boldsymbol{r} | \psi_j^{\theta} \rangle$ has an asymptotic form $\exp(-k_j^{\theta} r e^{i\theta})$, so that it becomes a normalizable L^2-type function for $0 \leq \theta < \pi/2$. Operating $C(\theta)$ on Eq. (11) leads to $(H_{\mathrm{B}}^{\theta} - \varepsilon) C(\theta) | \psi \rangle = 0$. Comparing this equation with the first equation of Eq. (19), one can find that $|\psi_j^{\theta}\rangle = C(\theta) | \psi_j \rangle$ and the eigenenergy ε_j^{θ} is independent of θ.

2. In general a resonance state $|\psi_\alpha^\theta\rangle$ has an energy $\varepsilon_\alpha^\theta = \hbar^2(k_\alpha^\theta e^{-i\phi})^2/(2\mu_{bc})$ with positive ϕ and k_α^θ. The state has an asymptotic form $\exp(ik_\alpha^\theta r e^{i(\theta-\phi)})$, and hence it is a normalizable L^2-type function for $\theta > \phi$. Let us take the case that $\theta_1 > \theta > \phi$. Operating $C(\theta_1 - \theta)$ on the first equation of Eq. (19) and using $C(\theta_1) = C(\theta_1-\theta)C(\theta)$, we obtain $(H_{\rm B}^{\theta_1} - \varepsilon_\alpha^\theta)C(\theta_1-\theta)|\psi_\alpha^\theta\rangle = 0$. Identifying this equation with $(H_{\rm B}^{\theta_1} - \varepsilon_\alpha^{\theta_1})|\psi_\alpha^{\theta_1}\rangle = 0$ leads to the fact that $|\psi_\alpha^{\theta_1}\rangle = C(\theta_1-\theta)|\psi_\alpha^\theta\rangle$ and $\varepsilon_\alpha^\theta$ does not depend on θ if $\theta > \phi$.
3. The scattering state $|\psi^\theta(\boldsymbol{k})\rangle$ should be normalizable as $\langle\psi^\theta(\boldsymbol{k})|\psi^\theta(\boldsymbol{k}')\rangle = \delta(\boldsymbol{k} - \boldsymbol{k}')$. This means that the incident-wave part of the state is $\varphi_0 = (2\pi)^{-3/2}\exp(i\boldsymbol{k}\cdot\boldsymbol{r})$ and hence $\boldsymbol{k}$ has to be transformed by the scaling transformation as $\boldsymbol{k} \to \boldsymbol{k}e^{-i\theta}$. Therefore, the scattering state has an energy $\varepsilon^\theta(\boldsymbol{k}) = (\hbar\boldsymbol{k}e^{-i\theta})^2/(2\mu_{bc})$, so that $|\psi^\theta(\boldsymbol{k})\rangle \neq C(\theta)|\psi^{\theta=0}(\boldsymbol{k})\rangle$. Note that $\langle\boldsymbol{r}|C(\theta)|\psi^{\theta=0}(\boldsymbol{k})\rangle$ diverges at large r and thus not normalizable.

The scattering state $|\psi(\boldsymbol{k})\rangle$ at $\theta = 0$ satisfies the integral equation

$$|\psi(\boldsymbol{k})\rangle = |\varphi_0\rangle + \frac{1}{\varepsilon - H_{\rm B}}V_{bc}|\varphi_0\rangle = |\varphi_0\rangle + C^{-1}(\theta)\frac{1}{\varepsilon - H_{\rm B}^\theta}V_{bc}^\theta C(\theta)|\varphi_0\rangle, \qquad (21)$$

where use has been made of $C^{-1}(\theta)C(\theta) = 1$ in the second equality of Eq. (21) and $|\varphi_0\rangle$ describes the plane wave satisfying

$$[T_r - \varepsilon]\,|\varphi_0\rangle = 0. \qquad (22)$$

One can thus obtain the smoothing factor by

$$\langle\hat{\Phi}_i|\psi\rangle = \langle\hat{\Phi}_i|\varphi_0\rangle + \langle\hat{\Phi}_i|C^{-1}(\theta)\frac{1}{\varepsilon - H_{\rm B}^\theta}V_{\rm bc}^\theta C(\theta)|\varphi_0\rangle, \qquad (23)$$

if the full propagator $G^\theta = (\varepsilon - H_{\rm B}^\theta)^{-1}$ is given for positive ε.

The eigenstates $\{\psi_j^\theta, \psi_\alpha^\theta, \psi^\theta(\boldsymbol{k})\}$ are normalizable and can form the complete set.[23] Thus, the spectral representation of G^θ is given by

$$G^\theta = \sum_j |\psi_j^\theta\rangle\frac{1}{\varepsilon - \varepsilon_j^\theta}\langle\tilde{\psi}_j^\theta| + \sum_\alpha |\psi_\alpha^\theta\rangle\frac{1}{\varepsilon - \varepsilon_\alpha^\theta}\langle\tilde{\psi}_\alpha^\theta| + \int d\boldsymbol{k}|\psi^\theta(\boldsymbol{k})\rangle\frac{1}{\varepsilon - \varepsilon^\theta(\boldsymbol{k})}\langle\tilde{\psi}^\theta(\boldsymbol{k})|. \qquad (24)$$

In this representation, the resonance part (the second term of the right-hand side) is separated out from the scattering part (the third term). This is useful in investigating a role of the resonance channel on the elastic scattering and the breakup reactions.

The exact representation Eq. (24) is still complicated and not so useful in particular for systems more complicated than the two-body system. The full propagator $\langle\boldsymbol{r}|G^\theta|\boldsymbol{r}'\rangle(= G^\theta(\boldsymbol{r},\boldsymbol{r}'))$ satisfies the equation

$$\begin{aligned} G^\theta(\boldsymbol{r},\boldsymbol{r}') &= G_0^\theta(\boldsymbol{r},\boldsymbol{r}') + \int d\boldsymbol{r}'' G_0^\theta(\boldsymbol{r},\boldsymbol{r}'')V(e^{i\theta}\boldsymbol{r}'')G^\theta(\boldsymbol{r}'',\boldsymbol{r}') \\ &= G_0^\theta(\boldsymbol{r},\boldsymbol{r}') + \int d\boldsymbol{r}'' G_0^\theta(\boldsymbol{r},\boldsymbol{r}'')V(e^{i\theta}\boldsymbol{r}'')G_0^\theta(\boldsymbol{r}'',\boldsymbol{r}') + \cdots, \end{aligned} \qquad (25)$$

where

$$G_0^\theta(\boldsymbol{r},\boldsymbol{r}') = \langle\boldsymbol{r}|\frac{1}{\varepsilon - e^{-2i\theta}T_r}|\boldsymbol{r}'\rangle = -\frac{\mu_{bc}e^{2i\theta}}{2\pi\hbar^2}\frac{\exp[i\boldsymbol{k}\cdot|\boldsymbol{r}-\boldsymbol{r}'|e^{i\theta}]}{|\boldsymbol{r}-\boldsymbol{r}'|}. \qquad (26)$$

The free propagator $G_0^\theta(\boldsymbol{r},\boldsymbol{r}')$ is an exponentially damping function of $|\boldsymbol{r}-\boldsymbol{r}'|$ when $0<\theta\leq\pi$. Each term in the last form of Eq. (25) has this property. Therefore, the full propagator $G^\theta(\boldsymbol{r},\boldsymbol{r}')$ keeps the damping property. This assures that G^θ can be expressed by a superposition of L^2-type basis functions; namely,

$$G^\theta \approx \mathcal{P}^\theta G^\theta \mathcal{P}^\theta, \tag{27}$$

where the operator $\mathcal{P}^\theta$ represents the modelspace spanned by L^2-type basis functions. The most convenient representation of $\mathcal{P}^\theta$ is

$$\mathcal{P}^\theta = \sum_i |\,\Phi_i^\theta\rangle\langle\tilde{\Phi}_i^\theta|, \tag{28}$$

where the Φ_i^θ are eigenstates of $H_{\rm B}^\theta$ obtained by diagonalizing it with L^2-type basis functions:

$$\langle\tilde{\Phi}_i^\theta|H_{\rm B}^\theta|\Phi_{i'}^\theta\rangle = \varepsilon_i^\theta\delta_{ii'}. \tag{29}$$

One can find the approximate relation $\langle\tilde{\Phi}_i^\theta|G^\theta|\Phi_{i'}^\theta\rangle \approx (\varepsilon-\varepsilon_i^\theta)^{-1}\delta_{ii'}$, because

$$\delta_{ii'} = \langle\tilde{\Phi}_i^\theta|\Phi_{i'}^\theta\rangle = \langle\tilde{\Phi}_i^\theta|(\varepsilon-H_{\rm B}^\theta)G^\theta|\Phi_{i'}^\theta\rangle \approx \langle\tilde{\Phi}_i^\theta|(\varepsilon-H_{\rm B}^\theta)\mathcal{P}^\theta G^\theta|\Phi_{i'}^\theta\rangle = (\varepsilon-\varepsilon_i^\theta)\langle\tilde{\Phi}_i^\theta|G^\theta|\Phi_{i'}^\theta\rangle. \tag{30}$$

Using this approximate relation, we can obtain

$$G^\theta \approx \sum_{ii'} |\,\Phi_i^\theta\rangle\langle\tilde{\Phi}_i^\theta|G^\theta|\,\Phi_{i'}^\theta\rangle\langle\tilde{\Phi}_{i'}^\theta| = \sum_i |\,\Phi_i^\theta\rangle\frac{1}{\varepsilon-\varepsilon_i^\theta}\langle\tilde{\Phi}_i^\theta|. \tag{31}$$

This approximation has been applied to calculations of the continuum level density,[24] the scattering amplitude,[25] and the strength function of the electromagnetic transition.[19),20)] Validity of the approximate expression Eq. (31) in the CDCC framework will be justified, if the breakup S-matrix elements converge as the modelspace $\mathcal{P}^\theta$ is extended. Inserting Eq. (31) into Eq. (23), we obtain the equation for the smoothing factor:

$$\langle\hat{\Phi}_i|\psi\rangle \approx \langle\hat{\Phi}_i|\varphi_0\rangle + \sum_{i'i''}\frac{1}{\varepsilon-\varepsilon_{i'}^\theta}\langle\hat{\Phi}_i|C^{-1}(\theta)|\,\Phi_{i'}^\theta\rangle\langle\tilde{\Phi}_{i'}^\theta|V^\theta|\,\Phi_{i''}^\theta\rangle\langle\tilde{\Phi}_{i''}^\theta|C(\theta)|\varphi_0\rangle. \tag{32}$$

When the Gaussian basis functions are taken, the matrix elements $\langle\hat{\Phi}_i|C^{-1}(\theta)|\,\Phi_{i'}^\theta\rangle$ and $\langle\tilde{\Phi}_{i''}^\theta|C(\theta)|\varphi_0\rangle$ are analytically obtained. The matrix elements $\langle\tilde{\Phi}_{i'}^\theta|V^\theta|\,\Phi_{i''}^\theta\rangle$ are also easily obtained by making a single integral. Thus, the final formula Eq. (32) for the smoothing factor is quite useful.

§3. Test calculations

In this section, we test the validity of the smoothing formula, Eq. (32), in two cases of the ^{58}Ni(d,pn) reaction at 80 MeV and the ^{12}C(^{6}He,^{4}He^2n) reaction at 229.8 MeV. In the former, we have chosen d as a typical projectile with no resonance and treat it with the n+p model. In the latter, we have selected ^{6}He as a projectile

with resonance and for simplicity will treat it with the two-body ^2n+^{4}He model. In principle, ^{6}He should be treated as the three-body n+n+^{4}He system. However, since our interest is the 2^+ resonance in the present test, we use the dineutron (^2n+^{4}He) model that can describe the 2^+ resonance reasonably well.

The pseudostates $\hat{\Phi}_i$ in the CDCC calculations are constructed by diagonalizing $H_{\rm B}$ with the complex-range Gaussian basis functions, which are found to effectively describe breakup processes.[10] The explicit forms of the basis functions are

$$\phi^{\rm C}_{j\ell}(r) = r^\ell \exp\left[-(r/a_j)^2\right] \cos\left[\pi/2(r/a_j)^2\right], \tag{33}$$

$$\phi^{\rm S}_{j\ell}(r) = r^\ell \exp\left[-(r/a_j)^2\right] \sin\left[\pi/2(r/a_j)^2\right], \tag{34}$$

where $j = 1$–N. The range parameter a_j is taken as a geometric progression,

$$a_j = a_1(a_N/a_1)^{(j-1)/(N-1)}. \tag{35}$$

The number of the basis functions is $2N$ for each ℓ.

On the other hand, the complex scaled states Φ^θ_i are constructed by diagonalizing $H^\theta_{\rm B}$ with the conventional real-range Gaussian functions,

$$\phi_{j\ell}(r) = r^\ell \exp[-(r/a_j)^2],\ (j = 1\text{–}N), \tag{36}$$

which are valid for describing the approximate expression of G^θ as shown in Ref. 25).

3.1. ^{58}*Ni(d,pn) reaction at 80 MeV*

In the analysis of the ^{58}Ni(d,pn) reaction, we adopt the same model Hamiltonian as in the previous paper.[10] As for the breakup states of d we take $\ell = 0$ and 2 states and truncated them at $k = 1.5$ fm^{-1}. The range parameters of the complex-range Gaussian basis functions are ($2N = 40, a_1 = 1.0$ fm$, a_N = 30.0$ fm). In the calculation of the complex scaled states, we take ($N = 30, a_1 = 1.0$ fm$, a_N = 30.0$ fm) as the parameter set of the real-range Gaussian basis functions.

Figure 1 shows the squared moduli of breakup S-matrix elements as a function of k at the grazing total angular momentum $J = 17$; panels (a), (b), (c) and (d) correspond to (ℓ, L)=(0,17), (2,15), (2,17) and (2,19), respectively. Here L is the orbital angular momentum regarding $\mathbf{R}$. The open circles are the results calculated with the exact smoothing factors that are obtained by solving the two-body Schrödinger equation (11) numerically. The dotted, dashed, and solid lines correspond to results of the new smoothing method with CSM at the scaling angle $\theta = 5°$, $10°$, and $15°$, respectively. One can see that the results converge to the exact solution as the scaling angle θ increases.

3.2. ^{12}*C(^{6}He,^{4}He^2n) reaction at 229.8 MeV*

We take $\ell = 0$ and 2 breakup states and truncate them at k =1.5 fm^{-1}. We use the following interaction $V_{^2n\alpha}$ between ^2n and ^{4}He:

$$V_{^2{\rm n}\alpha}(r) = V_0 \exp\left[-(r/\beta)^2\right], \tag{37}$$

where the depth V_0 and the range β are determined so as to reproduce the ground-state and resonance energies of ^{6}He. As for the interactions between ^{12}C and each

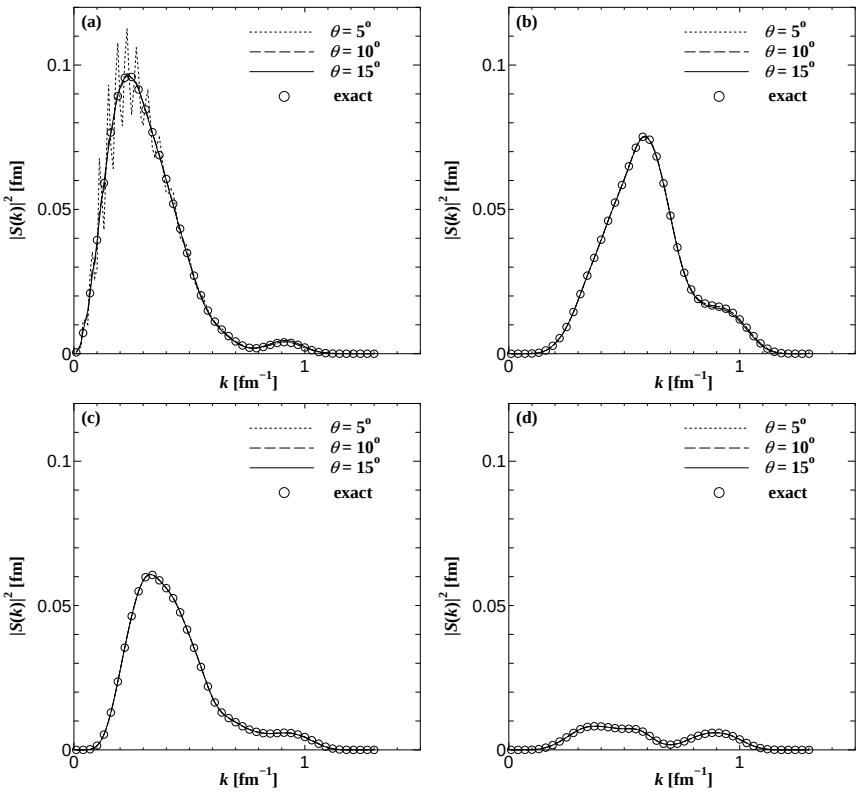

Fig. 1. The squared moduli of breakup S-matrix elements as a function of k at the grazing total angular momentum $J = 17$ for d+^{58}Ni scattering at 80 MeV. Panels (a), (b), (c), and (d) correspond to $(\ell, L) = (0, 17)$, (2,15), (2,17), and (2,19), respectively.

constituent of ^{6}He, i.e. ^{6}He and ^2n, we take the optical potentials of d+^{12}C[26] and ^{4}He+^{12}C.[27] The parameters of the complex-range Gaussian basis functions are ($2N = 40$, $a_1 = 1.0$ fm, $a_N = 30$ fm), and those of the real-range Gaussian basis functions are ($N = 30, a_1 = 1.0$ fm$, a_N = 30.0$ fm).

Figure 2 represents the squared moduli of the breakup S-matrix elements at the grazing total angular momentum $J = 25$. Panels (a), (b), (c), and (d) correspond to $(\ell, L) = (0, 25)$, (2,23), (2,25), and (2,27), respectively. The peaks shown in panels (b), (c), and (d) represent the 2^+ resonance of ^{6}He. The breakup S-matrix elements calculated with the simple smoothing formula Eq. (32) tend to the exact ones (the open circles) as θ increases. One can conclude from Fig. 2 that the smoothing formula is valid also for the resonance part.

§4. Summary

In this Letter, we present a practical formula for smoothing discrete breakup S-matrix elements calculated by CDCC. This smoothing procedure based on CSM can easily be performed by diagonalizing $H_{\rm B}$ and the complex scaled one $H_{\rm B}^{\theta}$ with the L^2-type bases such as the Gaussian basis function and inserting the eigenenergies

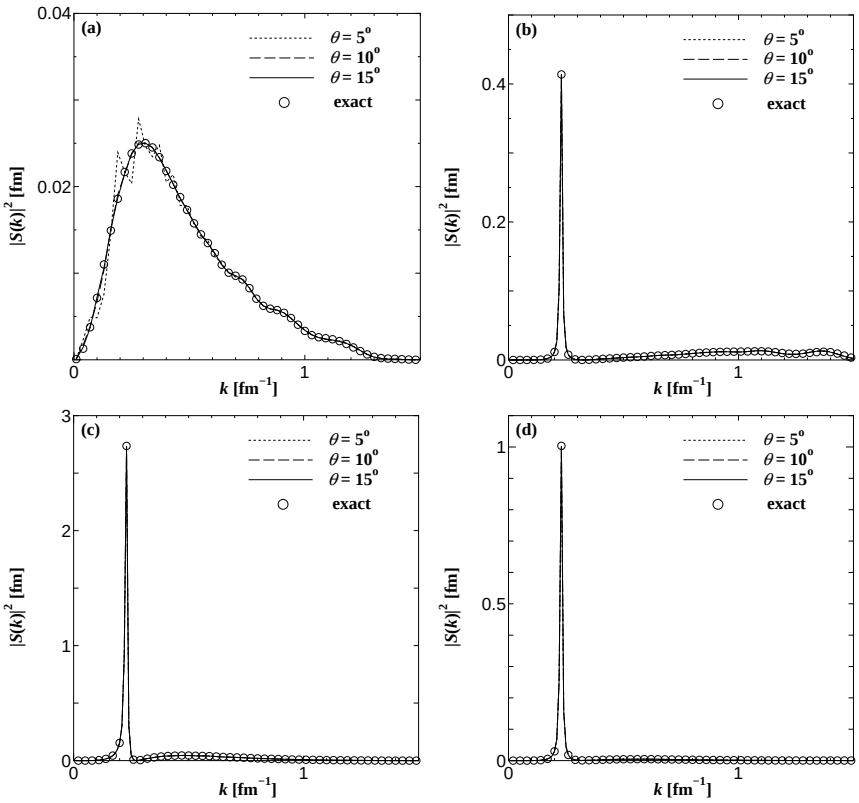

Fig. 2. The same as in fig. 1 but for ^{6}He+^{12}C scattering at 229.8 MeV. The corresponding grazing angular momentum is 25. Panels (a), (b), (c), and (d) represent $(\ell, L) = (0, 25)$, (2,23), (2,25), and (2,27), respectively.

and eigenstates into the formula. The validity of the formula is tested for two kinds of three-body breakup reactions, ^{58}Ni(d, pn) at 80 MeV and ^{12}C(^{6}He, ^{4}He^{2}n) at 229.8 MeV. For both the cases, the breakup S-matrix elements smoothed with the formula tend to the exact ones as the complex-scaling angle θ increases. Thus, the formula is accurate and practical. In a forthcoming paper, we will investigate the practicability of this formula for four-body breakup processes of three-body projectiles such as ^{6}He and apply this method for analyzing the experimental data[28)–30)] on the ^{6}He breakup reactions at lower and intermediate incident energies.

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